Comment On “On some Properties of Tribonacci Quaternion”

Gamaliel Cerda-Morales

Abstract


1 Corrections

Let

\[ Q_n = T_n + iT_{n+1} + jT_{n+2} + kT_{n+3}, \]

where \( T_n \) is the \( n \)-th Tribonacci number.

In [1], Akkus and Kızılaslan introduced the following identity (which is part of the identities 6 in [1])

\[ Q_n^2 - Q_{n-1}^2 = \tilde{U}_{n+1}\tilde{U}_{n-1}, \quad n \geq 2, \]

where

\[ \tilde{U}_n = U_n + iU_{n+1} + jU_{n+2} + kU_{n+3}, \]

with \( U_n = T_{n-1} + T_{n-2} \) if \( n \geq 2 \) and \( U_0 = U_1 = 0 \).

Key Words: Tribonacci numbers, Tribonacci quaternions, Tribonacci-Lucas quaternions, quaternion algebra.

2010 Mathematics Subject Classification: 11B37, 11R52, 11Y55.

Received: 20.06.2019
Accepted: 15.07.2019
However, the identity of this version is wrong. In fact, if $n = 2$, we have

\[
Q_2^2 - Q_1^2 = (1 + 2i + 4j + 7k)^2 - (1 + i + 2j + 4k)^2
\]

\[
= (-68 + 4i + 8j + 14k) - (-20 + 2i + 4j + 8k)
\]

\[
= -48 + 2i + 4j + 6k.
\]

and

\[
\tilde{U}_3\tilde{U}_1 = (2 + 3i + 6j + 11k)(i + 2j + 3k)
\]

\[
= -48 - 2i + 6j + 6k.
\]

The correct version should be as follows. For $\tilde{U}_n$ to be defined as above and $n \in \mathbb{N}$, then

\[
Q_n^2 - Q_{n-1}^2 = \tilde{U}_{n+1}\tilde{U}_{n-1} + 2\left(T_{-(n+3)}i - (T_{-(n+2)} - T_{-(n+1)})j + T_{-(n+2)}k\right),
\]

(1.1)

where $T_{-n}$ is the $n$-th Tribonacci negative number ($n \in \mathbb{N}$) and satisfies the recurrence relation

\[
T_{-(n+1)} = \begin{vmatrix}
    T_n & T_{n+1} \\
    T_{n-1} & T_n
\end{vmatrix} = T_n^2 - T_{n-1}T_{n+1}, \quad n \geq 1.
\]

There are several proofs of this famous quaternions, see for example [2, 3]. Actually, this identity plays a central role in proving one of the famous non-commutative properties of Tribonacci quaternions, say

\[
Q_{n+1}Q_n - Q_nQ_{n+1} = 2\left(T_{n+3}^2 - T_{n+2}T_{n+4}\right)i + 2\left(T_{n+1}T_{n+4} - T_{n+2}T_{n+3}\right)j
\]

\[
+ 2\left(T_{n+2}^2 - T_{n+1}T_{n+3}\right)k
\]

\[
= 2\left(T_{-(n+4)}i - (T_{-(n+3)} - T_{-(n+2)})j + T_{-(n+3)}k\right),
\]

(1.2)

with $n \geq 0$.

From Eqs. (1.1) and (1.2), we obtain

\[
Q_n^2 - Q_{n-1}^2 - Q_nQ_{n-1} + Q_{n-1}Q_n = \tilde{U}_{n+1}\tilde{U}_{n-1}, \quad n \geq 1.
\]

(1.3)

Similarly the identity 5 in [1]. Furthermore, one should note we should use the correct version of the identity (1.2) to obtain the conclusion.

References


Gamaliel CERDA-MORALES,
Institute of Mathematics,
Pontificia Universidad Católica de Valparaíso,
Blanco Viel 596, Cerro Barón, Valparaíso, Chile.
Email: gamaliel.cerda.m@mail.pucv.cl