INTEGRAL MEANS OF CERTAIN CLASSES OF ANALYTIC FUNCTIONS DEFINED BY DZIOK-SRIVASTAVA OPERATOR

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ABSTRACT. In this paper, we introduce the subclass $UT_{q,s}([\alpha_1]; \alpha, \beta)$ of analytic functions defined by Dziok-Srivastava operator. The object of the present paper is to determine the Silverman's conjecture for the integral means inequality to this class.

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1. Introduction

Let $A$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1.1)$$

which are analytic in the open unit disc $U = \{ z : |z| < 1 \}$. Let $K(\alpha)$ and $S^*(\alpha)$ denote the subclasses of $A$ which are, respectively, convex and starlike functions of order $\alpha$, $0 \leq \alpha < 1$. For convenience, we write $K(0) = K$ and $S^*(0) = S^*$ (see [18]). The Hadamard product (or convolution) $(f \ast g)(z)$ of the functions $f(z)$ and $g(z)$, that is, if $f(z)$ is given by (1.1) and $g(z)$ is given by $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, is defined by

$$(f \ast g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g \ast f)(z).$$

If $f$ and $g$ are analytic functions in $U$, we say that $f$ is subordinate to $g$, written $f \prec g$ if there exists a Schwarz function $w$, which (by definition) is analytic in $U$ with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in U$, such that $f(z) = g(w(z))$, $z \in U$.

For positive real parameters $\alpha_1, ..., \alpha_q$ and $\beta_1, ..., \beta_s$ ($\beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^\times$, $\mathbb{Z}_0^\times = 0, -1, -2, ...; j = 1, 2, ..., s$), the generalized hypergeometric function $\mathcal{F}_q(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z)$ is defined by
\[ qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \ldots (\alpha_q)_n}{(\beta_1)_n \ldots (\beta_s)_n n!} z^n \]

\[ (q \leq s + 1; s, q \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \mathbb{N} = \{1, 2, \ldots\}; \ z \in U), \]

where \((\theta)_n\) is the Pochhammer symbol defined in terms of the Gamma function \(\Gamma\), by

\[(\theta)_n = \frac{\Gamma(\theta + n)}{\Gamma(\theta)} = \left\{ \begin{array}{ll} 1 & (n = 0) \\ \theta(\theta + 1) \ldots (\theta + n - 1) & (n \in \mathbb{N}). \end{array} \right.\]

For the function \(h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) = z_qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)\), the Dziok-Srivastava linear operator (see [5] and [6]) \(H_{q,s}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; \ )\) : \(A \rightarrow A\), is defined by the Hadamard product as follows:

\[ H_{q,s}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s) f(z) = h(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) \ast f(z) = z + \sum_{n=2}^{\infty} \Psi_n(\alpha_1) a_n z^n \quad (z \in U), \quad (1.2) \]

where

\[ \Psi_n(\alpha_1) = \frac{(\alpha_1)_{n-1} \ldots (\alpha_q)_{n-1}}{(\beta_1)_{n-1} \ldots (\beta_s)_{n-1} (n-1)!}. \quad (1.3) \]

For brevity, we write

\[ H_{q,s}(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) f(z) = H_{q,s}(\alpha_1) f(z). \quad (1.4) \]

For \(0 \leq \alpha < 1, \beta \geq 0\) and for all \(z \in U\), let \(US_{q,s}([\alpha_1]; \alpha, \beta)\) denote the subclass of \(A\) consisting of functions \(f(z)\) of the form (1.1) and satisfying the analytic criterion

\[ \text{Re} \left\{ \frac{H_{q,s}(\alpha_1) f(z)}{z(H_{q,s}(\alpha_1) f(z))'} - \alpha \right\} > \beta \left| \frac{H_{q,s}(\alpha_1) f(z)}{z(H_{q,s}(\alpha_1) f(z))'} - 1 \right|. \quad (1.5) \]

Denote by \(T\) the subclass of \(A\) consisting of functions of the form:

\[ f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0), \quad (1.6) \]

which are analytic in \(U\). We define the class \(UT_{q,s}([\alpha_1]; \alpha, \beta)\) by:

\[ UT_{q,s}([\alpha_1]; \alpha, \beta) = US_{q,s}([\alpha_1]; \alpha, \beta) \cap T. \quad (1.7) \]
We note that for suitable choices of $q, s, \alpha$ and $\beta$, we obtain the following subclasses studied by various authors.

(1) For $q = 2$ and $s = \alpha_1 = \alpha_2 = \beta_1 = 1$ in (1.5), the class $UT_{2,1}(\{1\}; \alpha, \beta)$ reduces to the class $ST(\alpha, \beta)$

$$
\left\{ f \in T : \text{Re}\left\{ \frac{f(z)}{zf'(z)} - \alpha \right\} > \beta \left| \frac{f(z)}{zf'(z)} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, z \in U \right\}
$$

and the class $ST(\alpha, 0) = ST(\alpha)$ is the family of functions $f(z) \in T$ which satisfy the following condition (see [7] and [19])

$$
ST(\alpha) = \text{Re}\left\{ \frac{f(z)}{zf'(z)} \right\} > \alpha \quad (0 \leq \alpha < 1);
$$

(2) For $q = 2, s = 1, \alpha_1 = a (a > 0), \alpha_2 = 1$ and $\beta_1 = c (c > 0)$ in (1.5), the class $UT_{2,1}(\{a, 1; c\}; \alpha, \beta)$ reduces to the class $T(a; c; \alpha, \beta)$

$$
\left\{ f \in T : \text{Re}\left\{ \frac{L(a,c)f(z)}{z(L(a,c)f(z))} - \alpha \right\} > \beta \left| \frac{L(a,c)f(z)}{z(L(a,c)f(z))} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, z \in U \right\},
$$

where $L(a, c)$ is the Carlson - Shaffer operator (see [2]).

(3) For $q = 2, s = 1, \alpha_1 = \lambda + 1 (\lambda > -1)$ and $\alpha_2 = \beta_1 = 1$ in (1.5), the class $UT_{2,1}(\{\lambda + 1; \alpha, \beta\)$ reduces to the class $W_\lambda(\alpha, \beta)$

$$
\left\{ f \in T : \text{Re}\left\{ \frac{D^\lambda f(z)}{z(D^\lambda f(z))} - \alpha \right\} > \beta \left| \frac{D^\lambda f(z)}{z(D^\lambda f(z))} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, \lambda > -1, z \in U \right\} \text{ (see [11]),}
$$

where $D^\lambda (\lambda > -1)$ is the Ruscheweyh derivative operator (see [15]).

(4) For $q = 2, s = 1, \alpha_1 = v + 1 (v > -1), \alpha_2 = 1$ and $\beta_1 = v + 2$ in (1.5), the class $UT_{2,1}(\{v + 1; v + 2\}; \alpha, \beta)$ reduces to the class $\zeta T(v; \alpha, \beta)$

$$
\left\{ f \in T : \text{Re}\left\{ \frac{J_v f(z)}{z(J_v f(z))} - \alpha \right\} > \beta \left| \frac{J_v f(z)}{z(J_v f(z))} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, v > -1, z \in U \right\},
$$

where $J_v f(z)$ is the generalized Bernardi - Libera - Livingston operator (see [1], [8] and [10]).

(5) For $q = 2, s = 1, \alpha_1 = 2, \alpha_2 = 1$ and $\beta_1 = 2 - \mu (\mu \neq 2, 3, ...)$ in (1.5), the class $UT_{2,1}(\{2, 1; 2 - \mu\}; \alpha, \beta)$ reduces to the class $\mathcal{FT}(\mu; \alpha, \beta)$
In [16] Silverman found that the function $f_T$ subclasses of $f$ for all $I$ where $q$.

(8) For $q = 2, s = 1, \alpha_1 = \mu (\mu > 0), \alpha_2 = 1$ and $\beta_1 = \lambda + 1 (\lambda > -1)$ in (1.5), the class $UT_{2,1}([\mu, 1; \lambda + 1]; \alpha, \beta)$ reduces to the class $LT(\mu, \lambda; \alpha, \beta)$

$$= \left\{ f \in T : Re \left\{ \frac{I_{\lambda, \mu} f(z)}{z(I_{\lambda, \mu} f(z))'} - \alpha \right\} > \beta \left| \frac{I_{\lambda, \mu} f(z)}{z(I_{\lambda, \mu} f(z))'} - 1 \right|, -1 \leq \alpha < 1, \beta \geq 0, \mu > 0, \lambda > -1, z \in U \right\},$$

where $I_{\lambda, \mu} f(z)$ is the Choi-Saigo-Srivastava operator (see [4]);

(7) For $q = 2, s = 1, \alpha_1 = 2, \alpha_2 = 1$ and $\beta_1 = k + 1 (k > -1)$ in (1.5), the class $UT_{2,1}([2, 1; k + 1]; \alpha, \beta)$ reduces to the class $AT(k; \alpha, \beta)$

$$= \left\{ f \in T : Re \left\{ \frac{I_k f(z)}{z(I_k f(z))'} - \alpha \right\} > \beta \left| \frac{I_k f(z)}{z(I_k f(z))'} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, k > -1, z \in U \right\},$$

where $I_k f(z)$ is the Noor integral operator (see [12]);

(8) For $q = 2, s = 1, \alpha_1 = c (c > 0), \alpha_2 = \lambda + 1 (\lambda > -1)$ and $\beta_1 = a (a > 0)$ in (1.5), the class $UT_{2,1}([c, \lambda + 1; a]; \alpha, \beta)$ reduces to the class $FT(c, a; \lambda; \alpha, \beta)$

$$= \left\{ f \in T : Re \left\{ \frac{P^q(a, c) f(z)}{z(P^q(a, c) f(z))'} - \alpha \right\} > \beta \left| \frac{P^q(a, c) f(z)}{z(P^q(a, c) f(z))'} - 1 \right|, 0 \leq \alpha < 1, \beta \geq 0, c > 0, \lambda > -1, a > 0, z \in U \right\},$$

where $P^q(a, c) f(z)$ is the Cho-Kwon-Srivastava operator (see [3]).

In [16] Silverman found that the function $f_2 = z - \frac{z^2}{2}$ is often extremal over the family $T$. He applied this function to resolve his integral means inequality, conjectured and settled in [17]:

$$\int_0^{2\pi} \left| f(re^{i\theta}) \right|^\delta d\theta \leq \int_0^{2\pi} \left| f_2(re^{i\theta}) \right|^\delta d\theta,$$

for all $f \in T$, $\delta > 0$ and $0 < r < 1$. In [17], he also proved his conjecture for the subclasses $T^*(\alpha)$ and $C(\alpha)$ of $T$, where $C(\alpha)$ and $T^*(\alpha)$ are the classes of convex
and starlike functions of order $\alpha$, $0 \leq \alpha < 1$, respectively.
In this paper, we prove Silverman’s conjecture for functions in the class $US_{q,s}(\alpha_1; \alpha, \beta)$. Also by taking appropriate choices of the parameters $\alpha_1, \ldots, \alpha_q$ and $\beta_1, \ldots, \beta_s$, we obtain the integral means inequalities for several known as well as new subclasses of uniformly convex and uniformly starlike functions in $U$.

2. Coefficient estimates

Unless otherwise mentioned, we shall assume in the reminder of this paper that, the parameters $\alpha_1, \ldots, \alpha_q$ and $\beta_1, \ldots, \beta_s$ are positive real numbers, $-1 \leq \alpha < 1$, $\beta \geq 0$, $n \geq 2$, $z \in U$ and $\Psi_n(\alpha_1)$ is defined by (1.3).

**Theorem 1.** A function $f(z)$ of the form (1.6) is in the class $UT_{q,s}(\alpha_1; \alpha, \beta)$ if

$$\sum_{n=2}^{\infty} \frac{[2n - n(\alpha - \beta) - (\beta + 1)]\Psi_n(\alpha_1)}{1 - \alpha} a_n \leq 1 - \alpha. \quad (2.1)$$

**Proof.** Suppose that (2.1) is true. Since

$$\frac{[2n - n(\alpha - \beta) - (\beta + 1)]\Psi_n(\alpha_1)}{1 - \alpha} - n\Psi_n(\alpha_1) = \frac{(n-1)(1+\beta)}{1-\alpha} \Psi_n(\alpha_1) > 0,$$

we deduce

$$\sum_{n=2}^{\infty} n\Psi_n(\alpha_1) a_n < \sum_{n=2}^{\infty} \frac{[2n - n(\alpha - \beta) - (\beta + 1)]\Psi_n(\alpha_1)}{1 - \alpha} a_n \leq 1.$$

It suffices to show that

$$\beta \left| \frac{H_{q,s}(\alpha_1)f(z)}{z(H_{q,s}(\alpha_1)f(z))'} - 1 \right| - \text{Re} \left( \frac{H_{q,s}(\alpha_1)f(z)}{z(H_{q,s}(\alpha_1)f(z))'} - 1 \right) \leq 1 - \alpha,$$

we have

$$\beta \left| \frac{H_{q,s}(\alpha_1)f(z)}{z(H_{q,s}(\alpha_1)f(z))'} - 1 \right| - \text{Re} \left( \frac{H_{q,s}(\alpha_1)f(z)}{z(H_{q,s}(\alpha_1)f(z))'} - 1 \right) \leq (1 + \beta) \left| \frac{H_{q,s}(\alpha_1)f(z)}{z(H_{q,s}(\alpha_1)f(z))'} - 1 \right| \leq (1 + \beta) \frac{\sum_{n=2}^{\infty} (n-1)\Psi_n(\alpha_1) a_n}{1 - \sum_{n=2}^{\infty} n\Psi_n(\alpha_1) a_n} < 1 - \alpha.$$
This completes the proof of Theorem 1.

Unfortunately, the converse of the above Theorem 1 is not true. So we define the subclass $T_{q,s}(\{\alpha_1\};\alpha,\beta)$ of $UT_{q,s}(\{\alpha_1\};\alpha,\beta)$ consisting of functions $f(z)$ which satisfy (2.1).

**Remark 1.** Putting $q = 2, s = 1, \beta = 0$ and $\alpha_1 = \alpha_2 = \beta_1 = 1$ in Theorem 1, we will obtain the result obtained by Yamakawa [19, Lemma 2.1, with $n = p = 1$].

**Corollary 1.** Let the function $f(z)$ defined by (1.6) be in the class $T_{q,s}(\{\alpha_1\};\alpha,\beta)$, then

$$a_n \leq \frac{(1 - \alpha)}{\left[2n - n(\alpha - \beta) - (\beta + 1)\right]\Psi_n(\alpha_1)} \quad (n \geq 2). \quad (2.2)$$

The result is sharp for the function

$$f(z) = z - \frac{(1 - \alpha)}{\left[2n - n(\alpha - \beta) - (\beta + 1)\right]\Psi_n(\alpha_1)} z^n \quad (n \geq 2). \quad (2.3)$$

Putting $q = 2, s = 1, \alpha_1 = \lambda + 1(\lambda > -1)$ and $\alpha_2 = \beta_1 = 1$ in Theorem 1, we obtain the following corollary.

**Corollary 2.** A function $f(z)$ of the form (1.6) is in the class $W_\lambda(\alpha,\beta)$ if

$$\sum_{n=2}^{\infty} \left[2n - n(\alpha - \beta) - (\beta + 1)\right]\left(\frac{\lambda + 1}{n - 1}\right)^{n-1} a_n \leq 1 - \alpha.$$  

**Remark 2.** The result in Corollary 2 corrects the result obtained by Najafzadeh and Kulkarni [11, Lemma 1.1].

3. **Integral Means**

**Lemma 1 [9].** If the functions $f$ and $g$ are analytic in $U$ with $g < f$, then for $\delta > 0$ and $0 < r < 1$,

$$\int_0^{2\pi} \left|g(re^{i\theta})\right|^{\delta} d\theta \leq \int_0^{2\pi} \left|f(re^{i\theta})\right|^{\delta} d\theta.$$

Applying Theorem 1 and Lemma 1 we prove the following theorem.

**Theorem 2.** Suppose $f(z) \in T_{q,s}(\{\alpha_1\};\alpha,\beta), \delta > 0$, the sequence $\{\Psi_n(\alpha_1)\}$ $(n \geq 2)$ is non-decreasing and $f_2(z)$ is defined by:

$$f_2(z) = z - \frac{1 - \alpha}{(3 - 2\alpha + \beta)\Psi_2(\alpha_1)} z^2, \quad (3.1)$$

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then for \( z = r e^{i\theta}, \) \( 0 < r < 1, \) we have

\[
\int_0^{2\pi} |f(re^{i\theta})|^\delta \, d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\delta \, d\theta. \tag{3.2}
\]

**Proof.** For \( f(z) \) of the form (1.6), (3.2) is equivalent to proving that

\[
\int_0^{2\pi} \left| 1 - \sum_{n=2}^{\infty} a_n z^{n-1} \right|^\delta \, d\theta \leq \int_0^{2\pi} \left| 1 - \frac{(1 - \alpha)}{(3 - 2\alpha + \beta)\Psi_2(\alpha_1)} z \right|^\delta \, d\theta.
\]

By using Lemma 1, it suffices to show that

\[
1 - \sum_{n=2}^{\infty} a_n z^{n-1} \prec 1 - \frac{(1 - \alpha)}{(3 - 2\alpha + \beta)\Psi_2(\alpha_1)} z. \tag{3.3}
\]

Setting

\[
1 - \sum_{n=2}^{\infty} a_n z^{n-1} = 1 - \frac{(1 - \alpha)}{(3 - 2\alpha + \beta)\Psi_2(\alpha_1)} w(z), \tag{3.4}
\]

and using (2.1) and the hypotheses \( \{\Psi_n(\alpha_1)\} \) (\( n \geq 2 \)) is non-decreasing, we obtain

\[
|w(z)| = \left| \frac{(3 - 2\alpha + \beta)\Psi_2(\alpha_1) \sum_{n=2}^{\infty} a_n z^{n-1}}{(1 - \alpha)} \right|
\leq |z| \sum_{n=2}^{\infty} \frac{(3 - 2\alpha + \beta)\Psi_2(\alpha_1) a_n}{(1 - \alpha)}
\leq |z| \sum_{n=2}^{\infty} \frac{[2n - n(\alpha - \beta) - (\beta + 1)]\Psi_n(\alpha_1) a_n}{(1 - \alpha)}
\leq |z|.
\]

This completes the proof of Theorem 2.

Putting \( q = 2 \) and \( s = \alpha_1 = \alpha_2 = \beta_1 = 1 \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 3.** If \( f(z) \in ST(\alpha, \beta, \delta > 0) \), then the assertion (3.2) holds true, where

\[
f_2(z) = z - \frac{(1 - \alpha)}{(3 - 2\alpha + \beta)} z^2.
\]

Putting \( \beta = 0 \) in Corollary 3, we obtain the following corollary:

**Corollary 4.** If \( f(z) \in ST(\alpha, \delta > 0) \), then the assertion (3.2) holds true, where
\[ f_2(z) = z - \frac{(1 - \alpha)}{(3 - 2\alpha)} z^2. \]

Putting \( q = 2, \ s = 1, \ \alpha_1 = a \ (a > 0), \ \alpha_2 = 1 \) and \( \beta_1 = c \ (c > 0) \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 5.** If \( f(z) \in LT(a, c; \alpha, \beta), \delta > 0, \) then the assertion (3.2) holds true, where

\[ f_2(z) = z - \frac{(1 - \alpha)c}{(3 - 2\alpha + \beta)} z^2. \]

Putting \( q = 2, \ s = 1, \ \alpha_1 = \lambda + 1 \ (\lambda > -1), \ \alpha_2 = 1 \) and \( \beta_1 = 1 \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 6.** If \( f(z) \in W_\lambda(\alpha, \beta), \delta > 0, \) then the assertion (3.2) holds true, where

\[ f_2(z) = z - \frac{(1 - \alpha)}{(3 - 2\alpha + \beta)(\lambda + 1)} z^2. \]

Putting \( q = 2, \ s = 1, \ \alpha_1 = v + 1 \ (v > -1), \ \alpha_2 = 1 \) and \( \beta_1 = v + 2 \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 7.** If \( f(z) \in \zeta T(v; \alpha, \beta), \delta > 0, \) then the assertion (3.2) holds true, where

\[ f_2(z) = z - \frac{(1 - \alpha)(v + 2)}{(3 - 2\alpha + \beta)(v + 1)} z^2. \]

Putting \( q = 2, \ s = 1, \ \alpha_1 = 2, \ \alpha_2 = 1 \) and \( \beta_1 = 2 - \mu \ (\mu \neq 2, 3, \ldots) \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 8.** If \( f(z) \in FT(\mu; \alpha, \beta), \delta > 0, \) then the assertion (3.2) holds true, where

\[ f_2(z) = z - \frac{(1 - \alpha)(2 - \mu)}{2(3 - 2\alpha + \beta)} z^2. \]

Putting \( q = 2, \ s = 1, \ \alpha_1 = \mu(\mu > 0), \ \alpha_2 = 1 \) and \( \beta_1 = \lambda + 1(\lambda > -1) \) in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 9.** If \( f(z) \in LT(\mu, \lambda; \alpha, \beta), \delta > 0, \) then the assertion (3.2) holds true, where

\[ f_2(z) = z - \frac{(1 - \alpha)(\lambda + 1)}{\mu(3 - 2\alpha + \beta)} z^2. \]
Putting $q = 2$, $s = 1$, $\alpha_1 = 2$, $\alpha_2 = 1$ and $\beta_1 = k + 1(k > -1)$ in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 10.** If $f(z) \in AT(k; \alpha, \beta), \delta > 0$, then the assertion (3.2) holds true, where

$$f_2(z) = z - \frac{(1 - \alpha)(k + 1)}{2(3 - 2\alpha + \beta)}z^2.$$

Putting $q = 3$, $s = 2$, $\alpha_1 = c$, $\alpha_2 = \lambda + 1$ and $\beta_1 = a$ in Theorems 1 and 2, respectively, we obtain the following corollary:

**Corollary 11.** If $f(z) \in FT(c, \lambda; a; \alpha, \beta), \delta > 0$, then the assertion (3.2) holds true, where

$$f_2(z) = z - \frac{a(1 - \alpha)}{c(\lambda + 1)(3 - 2\alpha + \beta)}z^2.$$

**References**


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