COEFFICIENT ESTIMATES FOR A UNIFICATION OF SOME SUBCLASSES OF ANALYTIC AND BI-UNIVALENT FUNCTIONS OF MA-MINDA TYPE

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Abstract. In the present investigation, we consider a new general subclass $N_{\Sigma}^{\mu}(\lambda, \gamma; \phi)$ of the class $\Sigma$ consisting of analytic and bi-univalent functions in the open unit disk $U$. For functions belonging to the class introduced here, we find estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. Several connections to some of the earlier known results are also pointed out.

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1. Introduction, Definitions and Preliminaries

Let $A$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k,$$

which are analytic in the open unit disk

$$U = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

It is well-known that if $f(z)$ is an analytic univalent function from a domain $D_1$ onto a domain $D_2$, then the inverse function $g(z)$ defined by

$$g(f(z)) = z \quad (z \in D_1)$$

is an analytic and univalent mapping from $D_2$ to $D_1$. Moreover, by the familiar Koebe One-Quarter Theorem (see [3]), we know that the image of $U$ under every
function \( f \in S \) contains a disk of radius \( \frac{1}{4} \). Therefore, every univalent function \( f \in U \) has an inverse \( f^{-1} \) that satisfies the following conditions:

\[
f^{-1}(f(z)) = z \quad (z \in U)
\]

and

\[
f^{-1}(f(w)) = w \quad \left( w < r_0(f); \ r_0(f) \geq \frac{1}{4} \right).
\]

The inverse of the function \( f(z) \) has a series expansion in some disk about the origin of the form:

\[
f^{-1}(w) = w + \rho_2 w^2 + \rho_3 w^3 + \cdots.
\]

(2)

The inverse of the Koebe function provides the best bound for all \( |\rho_k| \) in (2) (see [8, 12]).

An univalent function \( f(z) \) in a neighborhood of the origin and its inverse \( f^{-1}(w) \) satisfy the following condition:

\[
f \left( f^{-1}(w) \right) = w
\]

or, equivalently,

\[
w = f^{-1}(w) + a_2 \left[ f^{-1}(w) \right]^2 + a_3 \left[ f^{-1}(w) \right]^3 + \cdots.
\]

(3)

Using (1) and (2) in (3), we obtain

\[
g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots.
\]

(4)

A function \( f \in A \) is said to be bi-univalent in \( U \) if both \( f(z) \) and \( f^{-1}(z) \) are univalent in \( U \). We denote by \( \Sigma \) the class of bi-univalent functions in \( U \) given by (1).

It is worth noting that the familiar Koebe function is not a member of \( \Sigma \) since it maps the unit disk \( U \) univalently onto the entire complex plane minus a slit along the line \( -\frac{1}{4} \) to \( -\infty \). Thus, the image of the domain does not contain the unit disk \( U \).

An analytic function \( f \) is subordinate to an analytic function \( g \), written \( f(z) \prec g(z) \), provided there is an analytic function \( w \) defined on \( U \) with \( w(0) = 0 \) and \( |w(z)| < 1 \) satisfying \( f(z) = g(w(z)) \). Ma and Minda [9] unified various subclasses of starlike and convex functions for which either of the quantity \( \frac{zf''(z)}{f'(z)} \) or \( 1 + zf''(z) \) is subordinate to a more general superordinate function. To this end, they considered an analytic function \( \phi \) with positive real part in the unit disk \( U \) such that \( \phi(0) = 1 \),

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\( \phi'(0) > 0 \), and \( \phi \) maps \( \mathbb{U} \) onto a region starlike with respect to 1 and symmetric with respect to the real axis. The class of Ma-Minda starlike functions consists of functions \( f \in \mathcal{A} \) satisfying the subordination \( \frac{zf'(z)}{f(z)} < \phi(z) \). Similarly, the class of Ma-Minda convex functions consists of functions \( f \in \mathcal{A} \) satisfying the subordination \( 1 + \frac{zf''(z)}{f'(z)} < \phi(z) \).

We now introduce the following unification of some subclasses of bi-univalent functions of Ma-Minda type.

**Definition 1.** A function \( f \in \Sigma \) is said to be in the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \), \( \mu \geq 0 \), \( \lambda \geq 1 \) and \( 0 \leq \gamma \leq 1 \), if the following subordinations hold:

\[
(1 - \lambda) \left( \frac{(1 - \gamma)z + \gamma f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{(1 - \gamma)z + \gamma f(z)}{z} \right)^{\mu - 1} < \phi(z) \tag{5}
\]

and

\[
(1 - \lambda) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^{\mu - 1} < \phi(w), \tag{6}
\]

where the function \( g \) is given by (4).

A function in the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \) is called bi-starlike of Ma-Minda type. This class unifies the subclass \( \mathcal{N}_\Sigma^{\phi,\phi}(\lambda, \mu) \) introduced recently by Srivastava et al. [13] and the subclass \( \mathcal{S}_{\Sigma}^{a,1,a}(1, \gamma, \phi) \) investigated by Peng et al. [11]. These subclasses are defined respectively as follows:

**Definition 2.** A function \( f \in \Sigma \) is said to be in the class \( \mathcal{N}_\Sigma^{\phi,\phi}(\lambda, \mu) \), \( \mu \geq 0 \) and \( \lambda \geq 1 \), if the following subordinations hold:

\[
(1 - \lambda) \left( \frac{f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{f(z)}{z} \right)^{\mu - 1} < \phi(z) \tag{7}
\]

and

\[
(1 - \lambda) \left( \frac{g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{g(w)}{w} \right)^{\mu - 1} < \phi(w), \tag{8}
\]

where the function \( g \) is given by (4).

**Definition 3.** A function \( f \in \Sigma \) is said to be in the class \( \mathcal{S}_{\Sigma}^{a,1,a}(1, \gamma, \phi) \), \( 0 \leq \gamma \leq 1 \), if the following subordinations hold:

\[
\frac{zf'(z)}{(1 - \gamma)z + \gamma f(z)} < \phi(z) \tag{9}
\]
and
\[
\left( \frac{wg'(w)}{(1-\gamma)w + \gamma g(w)} \right) < \phi(w), \tag{10}
\]
where the function \( g \) is given by (4).

It is easy to see that setting \( \gamma = 1 \) in Definition 1 leads us to Definition 2 and putting \( \mu = 0 \) and \( \lambda = 1 \) in Definition 1 leads us to Definition 3.

We shall mention that by suitably choosing \( \phi(z) \), the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \) reduces to interesting and important special cases. Let us give some examples.

**Example 1.** If we set \( \phi(z) = \frac{1 + Az}{1 + Bz} \), \(-1 \leq B < A \leq 1\), then the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \equiv \mathcal{N}_\Sigma(\lambda, \gamma; A, B) \) which is defined as \( f \in \Sigma \),
\[
(1 - \lambda) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^{\mu-1} < \frac{1 + Az}{1 + Bz}, \tag{11}
\]
and
\[
(1 - \lambda) \left( \frac{(1-\gamma)w + \gamma g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{(1-\gamma)w + \gamma g(w)}{w} \right)^{\mu-1} < \frac{1 + Aw}{1 + Bw}, \tag{12}
\]
where the function \( g \) is given by (4).

**Example 2.** Letting \( \phi(z) = \frac{1 + (1-2\beta)z}{1 - z} \), \( 0 \leq \beta < 1 \), then the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \equiv \mathcal{N}_\Sigma(\lambda, \gamma; \beta) \) which is defined as \( f \in \Sigma \),
\[
\Re \left( (1 - \lambda) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^{\mu-1} \right) > \beta, \tag{13}
\]
and
\[
\Re \left( (1 - \lambda) \left( \frac{(1-\gamma)w + \gamma g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{(1-\gamma)w + \gamma g(w)}{w} \right)^{\mu-1} \right) > \beta, \tag{14}
\]
where the function \( g \) is given by (4).

**Example 3.** If we put \( \phi(z) = \left( \frac{1 + z}{1 - z} \right)^\alpha \), \( 0 < \alpha \leq 1 \), then the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \equiv \mathcal{N}_\Sigma(\lambda, \gamma; \alpha) \) which is defined as \( f \in \Sigma \),
\[
\left| \arg \left( (1 - \lambda) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{(1-\gamma)z + \gamma f(z)}{z} \right)^{\mu-1} \right) \right| < \frac{\alpha \pi}{2}, \tag{15}
\]
and
\[ \left| \arg \left( (1 - \lambda) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^{\mu-1} \right) \right| < \frac{\alpha \pi}{2}, \]
(16)
where the function \( g \) is given by (4).

In 1967, Lewin [6] investigated the class \( \Sigma \) and showed that \( |a_2| < 1.51 \). Subsequently, Brannan and Clunie [1] conjectured that \( |a_2| \leq \sqrt{2} \). On the other hand, Netanyahu [10] showed that
\[ \max_{f \in \Sigma} |a_2| = \frac{4}{3}. \]

Afterwards in 1981, Styer and Wright [18] showed that there exist functions \( f(z) \in \Sigma \) for which \( |a_2| > \frac{4}{3} \). The best known estimate for functions in \( \Sigma \) has been obtained in 1984 by Tan [19], that is, \( |a_2| \leq 1.485 \). The coefficient estimate problem involving the bound of \( |a_n| \ (n \in \mathbb{N} \setminus \{1, 2\}) \) for each \( f \in \Sigma \) given by (1) is still an open problem.

Recently, many researchers [4, 5, 7, 14, 15, 16, 20, 21], following the work of Brannan and Taha [2], introduced and investigated a lot of interesting subclasses of the bi-univalent function class \( \Sigma \) and they obtained non-sharp estimates of the first two Taylor-Maclaurin coefficients \( |a_2| \) and \( |a_3| \).

In this paper, we derive estimates on the initial coefficients \( |a_2| \) and \( |a_3| \) for functions belonging to the unifying subclass \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \) of \( \Sigma \). Several connections to earlier known results are made.

The following lemma [3] will be required in order to derive our main results.

**Lemma 1.** If \( h \in \mathcal{P} \), then \( |c_k| \leq 2 \) for each \( k \in \mathbb{N} \), where \( \mathcal{P} \) is the family of all functions \( h \), analytic in \( \mathbb{U} \), for which
\[ \Re(h(z)) > 0, \quad (z \in \mathbb{U}), \]
where
\[ h(z) = 1 + c_1 z + c_2 z^2 + \cdots \quad (z \in \mathbb{U}). \]

**2. Coefficient Bounds for the functions class** \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \)

We begin by finding the estimates on the coefficient \( |a_2| \) and \( |a_3| \) for functions in the class \( \mathcal{N}_\Sigma^\mu(\lambda, \gamma; \phi) \).
Let $\phi$ be an analytic function with positive real part in the unit disk $\mathbb{U}$, satisfying $\phi(0) = 1$, $\phi'(0) > 0$, and $\phi(\mathbb{U})$ is symmetric with respect to the real axis. Such a function has a series expansion of the following form:

$$
\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots, \quad (B_1 > 0, \ z \in \mathbb{U}).
$$

(17)

Define the functions $p_1$ and $p_2$ in $\mathcal{P}$ given by

$$
p_1(z) = \frac{1 + u(z)}{1 - u(z)} = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots
$$

(18)

and

$$
p_2(z) = \frac{1 + v(z)}{1 - v(z)} = 1 + d_1 z + d_2 z^2 + d_3 z^3 + \cdots.
$$

(19)

It follows that

$$
u(z) = \frac{p_1(z) - 1}{p_1(z) + 1} = \frac{1}{2} \left( c_2 - \frac{c_1^2}{2} \right) z^2 + \cdots
$$

(20)

and

$$
v(z) = \frac{p_2(z) - 1}{p_2(z) + 1} = \frac{1}{2} \left( d_2 - \frac{d_1^2}{2} \right) z^2 + \cdots.
$$

(21)

Using (20) and (21) with (17) lead us to

$$
\phi(u(z)) = 1 + \frac{B_1 c_1}{2} z + \left\{ \frac{1}{2} \left( c_2 - \frac{c_1^2}{2} \right) B_1 + \frac{1}{4} c_1^2 B_2 \right\} z^2 + \cdots
$$

(22)

and

$$
\phi(v(z)) = 1 + \frac{B_1 d_1}{2} z + \left\{ \frac{1}{2} \left( d_2 - \frac{d_1^2}{2} \right) B_1 + \frac{1}{4} d_1^2 B_2 \right\} z^2 + \cdots.
$$

(23)

The following coefficient estimates hold for functions in the class $\mathcal{N}_\mu^\Sigma(\lambda, \gamma; \phi)$.

**Theorem 2.** Let $f(z) \in \mathcal{N}_\mu^\Sigma(\lambda, \gamma; \phi)$ be of the form (1). Then

$$
|a_2| \leq \frac{B_1 \sqrt{2B_1}}{\sqrt{B_1^2 \Omega(\mu, \lambda, \gamma) + 2(B_1 - B_2)(2\lambda - \gamma \lambda + \mu \gamma)^2}}
$$

(24)
and

\[ |a_3| \leq \frac{B_1^2}{(2\lambda - \gamma\lambda + \mu\gamma)^2} + \frac{B_1}{|3\lambda + \mu\gamma - \gamma\lambda|} \] (25)

where

\[ \Omega(\mu, \lambda, \gamma) = 4\mu\gamma\lambda - 6\gamma\lambda - 2\mu\gamma^2\lambda + 2\lambda\gamma^2 - \mu\gamma^2 + \mu^2\gamma^2 + 6\lambda + 2\mu\gamma \] (26)

and the coefficients \( B_1 \) and \( B_2 \) are given as in (17).

**Proof.** Let \( f \in N^\mu_\Sigma(\lambda, \gamma; \phi) \). Then there are analytic functions \( u, v : U \to \mathbb{U} \), with \( u(0) = v(0) = 0 \), satisfying

\[
(1 - \lambda) \left( \frac{(1 - \gamma)z + \gamma f(z)}{z} \right)^\mu + \lambda f'(z) \left( \frac{(1 - \gamma)z + \gamma f(z)}{z} \right)^{\mu-1} < \phi(z) \] (27)

and

\[
(1 - \lambda) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^\mu + \lambda g'(w) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^{\mu-1} < \phi(w) \] (28)

where \( g(w) := f^{-1}(w) \).

Now, equating the coefficients in (22), (23), (27) and (28), we obtain

\[
(2\lambda - \gamma\lambda + \mu\gamma) a_2 = \frac{B_1 c_1}{2}, \] (29)

\[
\frac{1}{2} \gamma(\mu - 1)(4\lambda + \mu\gamma - 2\gamma\lambda) a_2^2 + (3\lambda + \mu\gamma - \gamma\lambda) a_3
\]

\[
= \frac{1}{2} \left( c_2 - \frac{c_1^2}{2} \right) B_1 + \frac{1}{4} c_1^2 B_2, \] (30)

\[
-(2\lambda - \gamma\lambda + \mu\gamma) a_2 = \frac{B_1 d_1}{2}, \] (31)

and

\[
\frac{1}{2} \gamma(\mu - 1)(4\lambda + \mu\gamma - 2\gamma\lambda) a_2^2 + (3\lambda + \mu\gamma - \gamma\lambda) (2a_2^2 - a_3)
\]

\[
= \frac{1}{2} \left( c_2 - \frac{c_1^2}{2} \right) B_1 + \frac{1}{4} c_1^2 B_2, \] (32)
From (29) and (31), we find that
\[ c_1 = -d_1. \]  
(33)

Adding (30) and (32) and then using (33), we get
\[ (4\mu\gamma\lambda - 6\gamma\lambda - 2\mu\gamma^2\lambda + 2\lambda\gamma^2 - \mu\gamma^2 + \mu^2\gamma^2 + 6\lambda + 2\mu\gamma)a_2^2 \]
\[ = \frac{c_1^2}{2}(B_2 - B_1) + \frac{B_1}{2}(c_2 + d_2). \]  
(34)

For the sake of brevity, we will use the notation given in (26).

Now, using the notation defined above and combining (29) and (34), we obtain
\[ a_2^2 = \frac{B_1^2(c_2 + d_2)}{2 \left[B_1^2\Omega(\mu, \lambda, \gamma) + 2(B_1 - B_2)(2\lambda - \gamma\lambda + \mu\gamma)^2\right]}. \]  
(35)

Applying Lemma 1 for the coefficients \( c_2 \) and \( d_2 \), we find
\[ |a_2|^2 \leq \frac{2B_1^3}{B_1^2\Omega(\mu, \lambda, \gamma) + 2(B_1 - B_2)(2\lambda - \gamma\lambda + \mu\gamma)^2}, \]  
(36)

and thus
\[ |a_2| \leq \frac{B_1 \sqrt{2B_1}}{\sqrt{B_1^2\Omega(\mu, \lambda, \gamma) + 2(B_1 - B_2)(2\lambda - \gamma\lambda + \mu\gamma)^2}}, \]  
(37)

where \( \Omega(\mu, \lambda, \gamma) \) is given by (26).

Similarly, upon subtracting (32) from (30), we get
\[ 2(3\lambda + \mu\gamma - \gamma\lambda)(a_3 - a_2^3) = \frac{1}{2}B_1(d_2 - c_2). \]  
(38)

It follows from (29) and (38) that
\[ a_3 = \frac{B_1^2 c_1^2}{4(2\lambda - \gamma\lambda + \mu\gamma)^2} + \frac{B_1(d_2 - c_2)}{4(3\lambda + \mu\gamma - \gamma\lambda)}. \]  
(39)

Finally, applying Lemma 1 for the coefficients \( c_1, c_2 \) and \( d_2 \), we readily obtain
\[ |a_3| \leq \frac{B_1^2}{(2\lambda - \gamma\lambda + \mu\gamma)^2} + \frac{B_1}{|(3\lambda + \mu\gamma - \gamma\lambda)|}. \]  
(40)
3. Corollaries and Consequences

This section is devoted to the presentation of some interesting special cases of Theorem 1.

Let \( \phi(z) = \left( \frac{1 + z}{1 - z} \right)^{\alpha}, \) \( 0 < \alpha \leq 1 \) \( (B_1 = 2\alpha, \ B_2 = 2\alpha^2), \) in Theorem 1. Then, the class \( N_\Sigma^\mu(\lambda, \gamma; \phi) \) reduces to \( N_\Sigma^\mu(\lambda, \gamma; \alpha) \) given in Example 3 and thus, we get the following corollary:

**Corollary 3.** Let \( f(z) \in N_\Sigma^\mu(\lambda, \gamma; \alpha) \) be of the form (1). Then

\[
|a_2| \leq \frac{2\alpha}{\sqrt{|\alpha\Omega(\mu, \lambda, \gamma) - (\alpha - 1)(2\lambda - \gamma\lambda + \mu\gamma)|}} \quad (41)
\]

and

\[
|a_3| \leq \frac{4\alpha^2}{(2\lambda - \gamma\lambda + \mu\gamma)^2} + \frac{2\alpha}{|3\lambda + \mu\gamma - \gamma\lambda|} \quad (42)
\]

where \( \Omega(\mu, \lambda, \gamma) \) is given by (26).

Now, if we set \( \phi(z) = \frac{1 + (1 - 2\beta)z}{1 - z}, \) \( 0 \leq \beta < 1 \) \( (B_1 = B_2 = 2 - 2\beta), \) in Theorem 1, then the class \( N_\Sigma^\mu(\lambda, \gamma; \phi) \) reduces to \( N_\Sigma^\mu(\lambda, \gamma; \beta) \) given in Example 2 and then we obtain the following corollary:

**Corollary 4.**

\[
|a_2| \leq \sqrt{\frac{2(1 - \beta)}{|\Omega(\mu, \lambda, \gamma)|}} \quad (43)
\]

and

\[
|a_3| \leq \frac{4(1 - \beta)^2}{(2\lambda - \gamma\lambda + \mu\gamma)^2} + \frac{2(1 - \beta)}{|3\lambda + \mu\gamma - \gamma\lambda|} \quad (44)
\]

where \( \Omega(\mu, \lambda, \gamma) \) is given by (26).

Numerous other (presumably new) corollaries and consequences of our main result can also be deduced by specializing the different parameters involved in the class \( N_\Sigma^\mu(\lambda, \gamma; \phi) \). For example, letting \( \lambda = 1 \) in Theorem 1 leads us to the following corollary:
Corollary 5. Let \( f(z) \in \mathcal{N}_{\Sigma}^{\mu}(1, \gamma; \phi) \) be of the form (1). Then

\[
|a_2| \leq \frac{B_1 \sqrt{2B_1}}{\sqrt{|B_2^2 \left[ 6 + \gamma(\mu - 1) \left( \gamma(\mu - 2) + 6 \right) \right] + 2(B_1 - B_2)(2 - \gamma + \mu \gamma)^2|}}
\]  

(45)

and

\[
|a_3| \leq \frac{B_1^2}{(2 - \gamma + \mu \gamma)^2} + \frac{B_1}{(3 + \mu \gamma - \gamma)}
\]  

(46)

where the coefficients \( B_1 \) and \( B_2 \) are given as in (17).

The class \( \mathcal{N}_{\Sigma}^{\mu}(1, \gamma; \phi) \) is explicitly defined as follows:

Definition 4. A function \( f \in \Sigma \) is said to be in the class \( \mathcal{N}_{\Sigma}^{\mu}(1, \gamma; \phi) \), \( \mu \geq 0 \) and \( 0 \leq \gamma \leq 1 \), if the following subordinations hold:

\[
f'(z) \left( \frac{(1 - \gamma)z + \gamma f(z)}{z} \right)^{\mu - 1} \prec \phi(z)
\]  

(47)

and

\[
g'(w) \left( \frac{(1 - \gamma)w + \gamma g(w)}{w} \right)^{\mu - 1} \prec \phi(w),
\]  

(48)

where the function \( g \) is given by (4).

Obviously, by setting \( \gamma = 1 \) in Theorem 1, we recover the result obtained by Srivastava et al. [13]. Also, letting \( \mu = 0 \) and \( \lambda = 1 \) in Theorem 1, we find the result given recently by Peng et al. [11].

References


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