We consider a fourth order partial differential equation (PDE) which appeared in a classification of integrable PDEs of Boussinesq type (with a second order time derivative). Using the prolongation algebra method of Wahlquist-Estabrook, we obtain a $2 \times 2$ Lax pair for the PDE, and associate it to a Schrödinger operator with energy-dependent potential. By making a reduction to the stationary flow of a higher symmetry in the associated integrable hierarchy, we find a perturbation of a type (ii) Henon-Heiles system that has the weak Painlevé property i.e. its general solution has algebraic branching. Separating variables in this integrable Hamiltonian system, we obtain a class of solutions for the PDE that include the interaction of two solitons on a constant background. This is joint work with Caroline Verhoeven and Vladimir Novikov.