Most problems of modern Quantum and Statistical Physics are formulated in terms of Hamiltonian PDEs: the Schroedinger, Dirac, Maxwell, Yang-Mills equations, the coupled equations, etc. The nonlinear nature of the coupled equations is crucial for a consistent description of the fundamental physical phenomena: Bohr’s transitions to quantum stationary states, de Broglie’s wave particle duality, and the Maxwell-Boltzmann-Gibbs equilibrium statistics.

At the same time, it is this nonlinear nature that creates tremendous difficulties for a rigorous investigation of the fundamental phenomena. This fact leads to open mathematical problems for the nonlinear coupled PDEs, related to the Heisenberg Program that was formulated in 1961.

We suggest that these fundamental phenomena are intrinsic mathematical features of general nonlinear Hamiltonian PDEs. We state the related open problems and describe recent results in these directions.