The behaviour of nonsmooth systems have seen growing interest in recent years, motivated by models from industrial mechanics and electronics, from ecology, and from neuroscience. Local dynamics at a discontinuity can now be classified geometrically, using a small family of fundamental singularities and bifurcations. Moreover, these allow us to classify global dynamics in the form of sliding bifurcations. It also reveals more novel phenomena, such as discontinuity-induced explosions, which occur when determinism breaks down inside a well-defined nonsmooth flow. The geometry behind all of these suggests a fundamental link between nonsmooth systems, and the dynamics of (smooth) singularly perturbed systems, which we access through an approximation method called “pinching”.